

# QCD phase diagram from low energy effective models

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- Motivation and some old results
- The (axial)vector meson extended Polyakov-loop–const. quark–meson model\*
- Approximations used to solve the model
- Parametrization of the model
- Results:
  - $T$  dependence of curvature masses and condensates
  - $T$  and  $\mu_B$  dependence of various thermodynamical observables
  - $T - \mu_B$  and  $T - \rho_B$  phase diagrams, existence and location of CEP
- Conclusions

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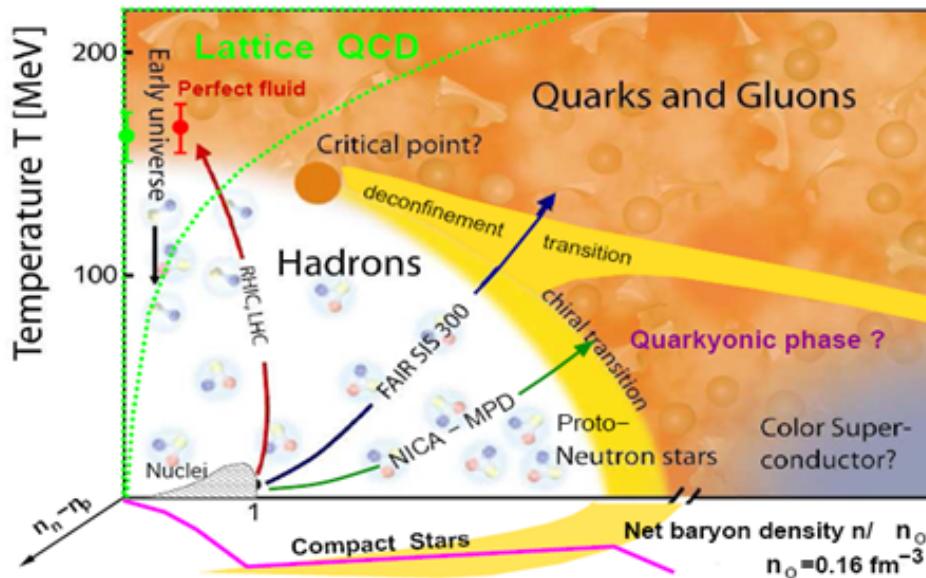
\* Based on: P. Kovács, Zs. Sz., Gy. Wolf, PRD93 (2016) 114014

# Motivation

Effective models helps revealing a reach phase structure at large  $\mu_B$ .

The success of an effective model depends on: d.o.f used, implemented resummation, parametrization of the model . . .

Phase diagram in the  $T - \mu_B - \mu_I$  space



- For  $\mu_B=0$  chiral phase transition at  $T_c \approx 153$  MeV [Aoki et al., PLB643 \(2006\) 46](#)  
[Bazavov et al., PRD85 \(2012\) 054503](#)
- The  $T$ -dependence of thermodynamical quantities like pressure, interaction measure, quark density is known from lattice only at  $\mu_B=0$ .
- Is there a CEP?
- At which  $\mu_B$  is the phase boundary for  $T=0$ ?

With only  $\bar{q}q$  states included, the vacuum parametrization of eL $\sigma$ M favors the assignment of  $f_0(1370)$  and  $f_0(1710)$  to the  $f^{L(ow)}$  and  $f^{H(igh)}$  states of the scalar nonet.

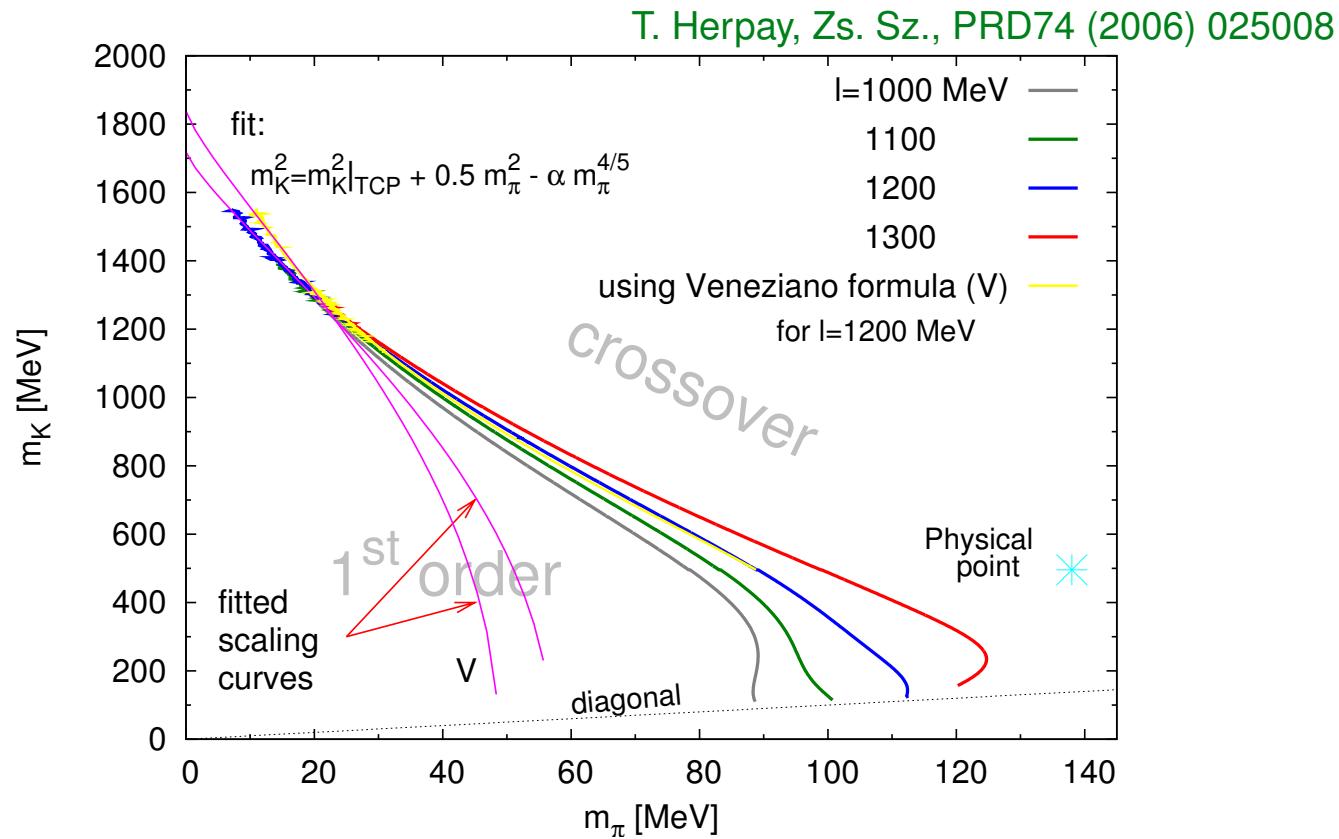
[D. Parganlija et al., PRD 87 \(2013\) 014011](#)

Given that  $\sigma$  is regarded as the radial excitation of the chiral OP, how a large  $m_{f_0^L}$  is compatible with a smooth crossover transition at  $T \approx 150$  MeV?

# Phase boundary with one-loop parametrization of the L $\sigma$ M

estimate for  $m_\pi = m_K$ :  $m_\pi^c \in (90, 130)$  MeV

location of TCP:  $m_K^{\text{TCP}} \in (1700, 1850)$  MeV  $\Rightarrow m_s = (13 - 15) \times m_s^{\text{phys}}$



Veneziano formula:  $m_\eta^2 = m_K^2 + \frac{1}{2} \Delta m_{\eta 0}^2 - \frac{1}{2} [(2m_K^2 - 2m_\pi^2 - \frac{1}{3} \Delta m_{\eta 0}^2)^2 + \frac{8}{9} \Delta m_{\eta 0}^4]^{1/2}$   
 $\Delta m_{\eta 0}^2$  non-perturbative gluonic contribution

$N_t = 6$  lattice result:  $m_\pi^c \lesssim 50$  MeV

Bazavov *et al.*, arXiv:1701.03548  
based on 6  $m_q$  values corresponding to  $m_\pi \in (80, 230)$  MeV

- 1-loop level parametrization uses the OPT of Chiku & Hatsuda, PRD58:076001 principle of minimal sensitivity:  $M_\pi^2 = iG^{-1}(p^2 = M_\pi^2) \Big|_{\text{1-loop}} \stackrel{!}{=} m_{\pi, \text{tree}}^2$

- 1-loop pole masses for  $\pi, K, \eta$  are used

$$\Sigma_\pi = \sum_{i=\pi, K, \eta, \eta'} \pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \pi + \sum_{i=a_0, \kappa, \sigma, f_0} \pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \pi + \sum_{i=a_0, \sigma, f_0} \pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \pi + \sum_{i=\eta, \eta'} \pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \pi + \sum_{i=\eta, \eta'} \pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \pi + \pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \pi + \frac{\pi \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \pi}{\Delta m^2}$$

$$\Sigma_K = \sum_{i=\pi, K, \eta, \eta'} K \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} K + \sum_{i=a_0, \kappa, \sigma, f_0} K \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} K + \sum_{i=a_0, \sigma, f_0} K \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} K + \sum_{i=\pi, \eta, \eta'} K \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} K + \frac{K \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} K}{\Delta m^2}$$

$$\Sigma_{\eta_{kl}} = \sum_{i=K, \eta, \eta'} k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} l + k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} l + \sum_{i=\sigma, f_0}^{j=\eta, \eta'} k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} l + k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} l + k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} l$$

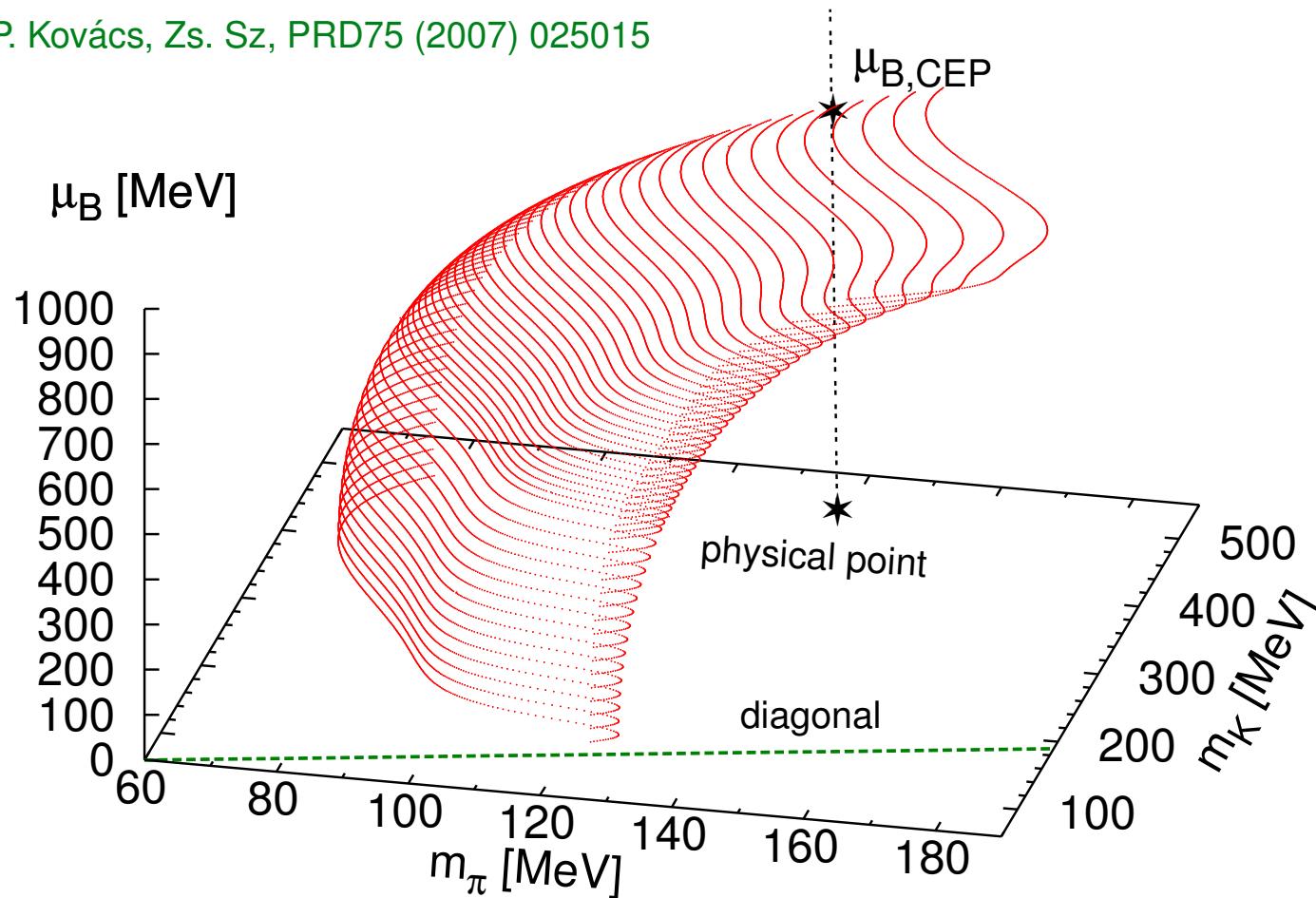
$$+ \delta_{kl} \left[ k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} l + \sum_{i=a_0, \sigma, f_0} k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} l + \frac{k \begin{array}{c} \diagup \\[-1ex] \diagdown \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} l}{\Delta m^2} \right]$$

- $m_\eta, f_\pi, f_K$  determined using  $SU(3)$  ChPT in the large  $N_c$ -limit

## Adding fermionic d.o.f. $\implies$ CQM model

- ChPT for baryons used to obtain the value of  $m_{u,s}$

P. Kovács, Zs. Sz, PRD75 (2007) 025015



The surface bends towards the physical point  $\implies$  existence of CEP

# Dependence of $\mu_B^{\text{CEP}}$ on the width of the susceptibility at $\mu = 0$

cf. P. Kovács, PhD thesis

Lattice results (fixed  $a$ ):

$$(\mu_B, T_c)^{\text{CEP}} = (725 \pm 35, 160 \pm 3.5) \text{ MeV}$$

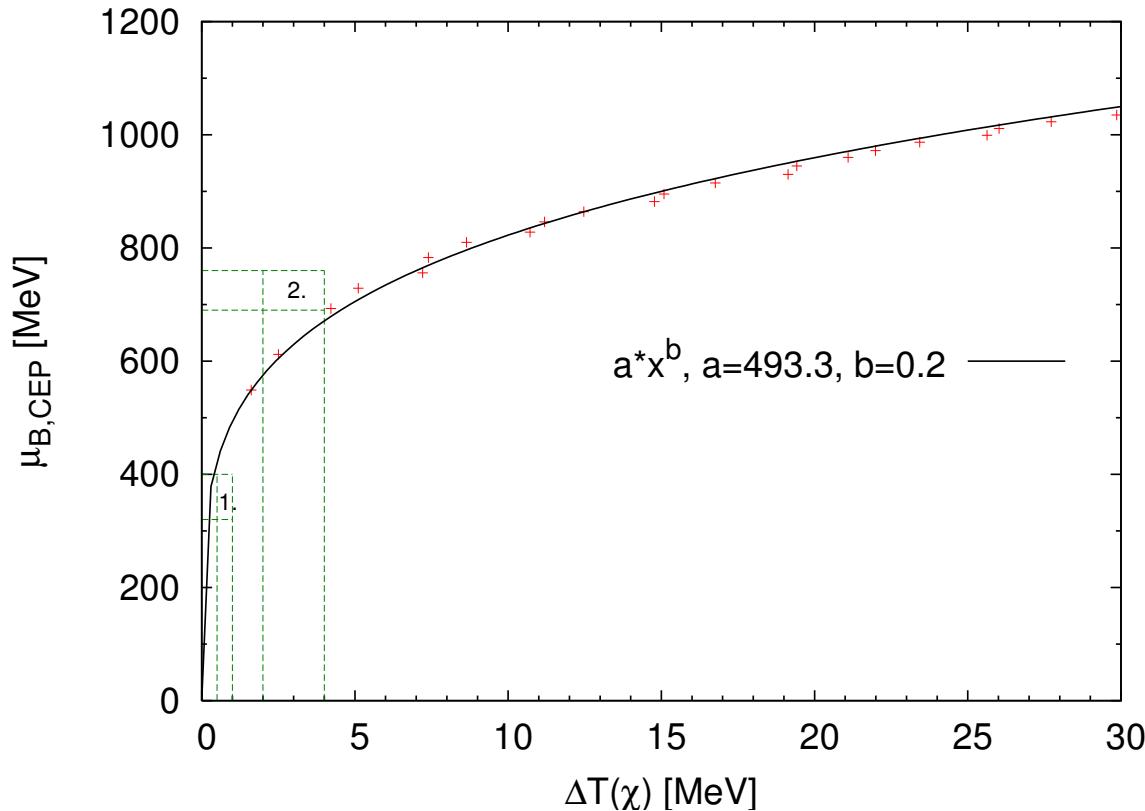
$$m_\pi \approx 2m_\pi^{\text{phys}}$$

Fodor & Katz, JHEP 0203:014, 2002

$$(\mu_B, T_c)^{\text{CEP}} = (360 \pm 40, 162 \pm 3) \text{ MeV}$$

$$m_\pi = m_\pi^{\text{phys}}$$

Fodor & Katz, JHEP 0404:050, 2004



☛ estimation by S. D. Katz:  $\Delta T_c(\chi_{\bar{\psi}\psi}) \approx 0.5 - 1 \text{ MeV}$  and  $\Delta T_c(\chi_{\bar{\psi}\psi}) \approx 2 - 4 \text{ MeV}$

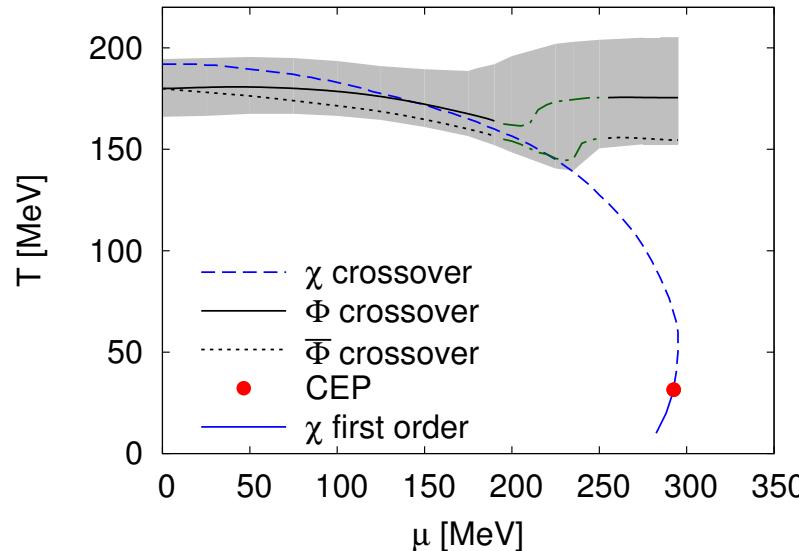
☛  $T_c(\chi_{\bar{\psi}\psi}) \approx 28 \text{ MeV}$  at the physical point in the continuum limit      Aoki *et al.* (2006)  
 ⇒ higher  $\mu_B^{\text{CEP}}$  can be expected in continuum limit

# Meson fluctuations

When using OPT the mesonic fluctuations where included at 1-loop level with the result that  $\frac{dT}{d\mu_B} < 0$  on the phase boundary  $\forall \mu_B > 0$

The inclusion of the mesonic fluctuation in FRG usually results in a change of sign in a given region of the phase boundary:  $\exists (\mu_B, T)$  for which  $\frac{dT}{d\mu_B} > 0$

Herbst *et al.*, PLB696 (2010) 58



Clausius-Clapeyron: along the 1<sup>st</sup> order transition line  $\frac{dT}{d\mu_B} = -\frac{\Delta\rho_B}{\Delta s}$ ,  $\Delta Q = Q_{\text{dense}} - Q_{\text{dilute}}$   
 $\implies \Delta s$  changes sign

# **Vector meson extended PQM model**

# Lagrangian

$\mathcal{L}$  constructed based on linearly realized global  $U(3)_L \times U(3)_R$  symmetry and its explicit breaking using:

1. matter fields:  $\Psi = (u, d, s)^\top$ ,  $M \equiv M_S + M_{PS} \equiv (S_a + i P_a) T_a$ ,  $T_a : U(3)$  generators  
 $R^\mu \equiv V^\mu - A^\mu \equiv (V_a^\mu - A_a^\mu) T_a$ ,  $L^\mu \equiv V^\mu + A^\mu \equiv (V_a^\mu + A_a^\mu) T_a$
2. external fields:  $H = H_0 T_0 + H_8 T_8$ ,  $\Delta = \Delta_0 T_0 + \Delta_8 T_8$ ,  $H_3 = \Delta_3 = 0 \Rightarrow$  exact  $SU(2)_V$  symmetry

$$\mathcal{L} = \mathcal{L}_{eL\sigma M} + \mathcal{L}_{qM}$$

$$\begin{aligned} \mathcal{L}_{eL\sigma M} &= \text{Tr}[(D_\mu M)^\dagger (D_\mu M)] - m_0^2 \text{Tr}(M^\dagger M) - \lambda_1 [\text{Tr}(M^\dagger M)]^2 - \lambda_2 \text{Tr}(M^\dagger M)^2 \\ &\quad + c_1 (\det M + \det M^\dagger) + \text{Tr}[\textcolor{blue}{H}(M + M^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ &\quad + \text{Tr} \left[ \left( \frac{m_1^2}{2} \mathbb{1} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ &\quad + \frac{h_1}{2} \text{Tr}(M^\dagger M) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[|L_\mu M|^2 + |MR_\mu|^2] + 2h_3 \text{Tr}(L_\mu M R^\mu M^\dagger) \end{aligned}$$

$$\mathcal{L}_{qM} = \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (M_S + i \gamma_5 M_{PS}) \Psi$$

$U(1)_A$  anomaly: for  $c_1 \neq 0$   $U(3)_L \times U(3)_R$  broken to  $SU(3)_V \times SU(3)_A \times U(1)_V$

Not all of the dim. 4 terms of D. Paganlja *et al.*, PRD 87, 014011 are used in  $\mathcal{L}_{eL\sigma M}$ .

Two more fields in the Lagrangian: electromagnetic  $A_e^\mu$  and gluon  $G_a^\mu$

$$D^\mu M = \partial^\mu M - ig_1(L^\mu M - MR^\mu) - ieA_e^\mu[T_3, M], \quad L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - (\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]) \\ D^\mu \Psi = \partial^\mu \Psi - iG^\mu \Psi \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a, \quad R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - (\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu])$$

- $A_e^\mu$  influences  $\Gamma_{a_1 \rightarrow \pi\gamma}$
- propagation of quarks on spatially constant temporal gluon background  
 $G_\mu = \delta_\mu^4 G_4$  leads with some further **approximations** to the **Polyakov loop** d.o.f.

In pure  $SU(3)$  thermal expect. value of traced Polyakov loop operator  $L(\vec{x}) = \mathcal{P} e^{i \int_0^\beta d\tau G_4(\tau, \vec{x})}$   
 $\Phi(\vec{x}) = \frac{1}{N_c} \langle \text{Tr}_c L(\vec{x}) \rangle$  and  $\bar{\Phi}(\vec{x}) = \frac{1}{N_c} \langle \text{Tr}_c \bar{L}(\vec{x}) \rangle$  signals breaking  
of the  $Z_{N_c}$  center symmetry at the deconfinement transition

low  $T$ ,  $\Phi(\vec{x}), \bar{\Phi}(\vec{x})=0 \rightarrow$  confined phase      high  $T$ ,  $\Phi(\vec{x}), \bar{\Phi}(\vec{x}) \neq 0 \rightarrow$  deconfined phase

**Polyakov gauge:**  $G_4$   $\tau$ -independent and is gauge rotated to a diagonal form in color space:  $G_4 = \phi_3 \lambda_3 + \phi_8 \lambda_8 \longrightarrow L = e^{i\beta G_4} = \text{diag}(a, b, c)$

Having  $G_4$  is like having an imaginary chemical potential for quarks:

$$D_q^{-1}(p) = \not{p} + m_q + \gamma_0(\mu_q - iG_4)$$

$\implies L = e^{i\beta G_4}$  appears when the partition function is calculated at lowest order  
**approximation:**  $\text{Tr}_c L \longrightarrow \Phi, \text{Tr}_c L^\dagger \longrightarrow \bar{\Phi}$  (w/o thermal average!)

# Particle content

Strange – non–strange ( $N - S$ ) basis

$$\xi_a \in (S_a, P_a, V_a^\mu, A_a^\mu, H_a, \Delta_a)$$

$$\xi_N = \frac{1}{\sqrt{3}} (\sqrt{2} \xi_0 + \xi_8), \quad \xi_S = \frac{1}{\sqrt{3}} (\xi_0 - \sqrt{2} \xi_8)$$

- **Vector** and **Axial-vector** meson nonets

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \omega_S \end{pmatrix}^\mu$$

$$\begin{aligned} \rho &\rightarrow \rho(770), K^* \rightarrow K^*(894) \\ \omega_N &\rightarrow \omega(782), \omega_S \rightarrow \phi(1020) \end{aligned}$$

$$A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$$\begin{aligned} a_1 &\rightarrow a_1(1230), K_1 \rightarrow K_1(1270) \\ f_{1N} &\rightarrow f_1(1280), f_{1S} \rightarrow f_1(1426) \end{aligned}$$

- **Scalar** ( $\sim \bar{q}_i q_j$ ) and **pseudoscalar** ( $\sim \bar{q}_i \gamma_5 q_j$ ) meson nonets

$$M_S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & \bar{K}_0^{*0} & \sigma_S \end{pmatrix}$$

$$\begin{aligned} \text{unknown assignment} \\ \text{mixing in the } \sigma_N - \sigma_S \text{ sector} \end{aligned}$$

$$M_{PS} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

$$\begin{aligned} \pi &\rightarrow \pi(138), K \rightarrow K(495) \\ \text{mixing: eigenvalues of } \eta, \eta' &\rightarrow \eta(548), \eta'(958) \end{aligned}$$

Spontaneous symmetry breaking:  $\sigma_{N/S}$  acquire nonzero expectation values  $\phi_{N/S}$   
 fields shifted by their expectation value:  $\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S}$

# Structure of scalar mesons below 2 GeV

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	$980 \pm 20$	$50 - 100$	$\pi\pi$ dominant
$a_0(1450)$	$1474 \pm 19$	$265 \pm 13$	$\pi\eta, \pi\eta', K\bar{K}$
$K_0^*(800) \equiv \kappa$	$682 \pm 29$	$547 \pm 24$	$K\pi$
$K_0^*(1430)$	$1425 \pm 50$	$270 \pm 80$	$K\pi$ dominant
$f_0(500) \equiv \sigma$	$400 - 550$	$400 - 700$	$\pi\pi$ dominant
$f_0(980)$	$980 \pm 20$	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	$1200 - 1500$	$200 - 500$	$\pi\pi \approx 250, K\bar{K} \approx 150$
$f_0(1500)$	$1505 \pm 6$	$109 \pm 7$	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	$1722 \pm 6$	$135 \pm 7$	$\pi\pi \approx 30, K\bar{K} \approx 71$

scalar  $\bar{q}q$  nonet content: 1  $a_0$ , 1  $K_0^*$ , and 2  $f_0$ s  $\implies$  40 possible assignments

result of a  $T = 0$  parametrization:  $a_0^{\bar{q}q} \rightarrow a_0(1450)$ ,  $K_0^{*,\bar{q}q} \rightarrow K_0^*(1430)$   
 $f_0^{L,\bar{q}q} \rightarrow f_0(1370)$ ,  $f_0^{H,\bar{q}q} \rightarrow f_0(1710)$

D. Parganlija *et al.*, PRD87, 014011

Considering **only  $\bar{q}q$  states** is unrealistic because most probably scalars are mixtures of  $\bar{q}q$ , tetraquarks and glueballs, nevertheless we will do this here.

$f_0(500)$ ,  $f_0(980)$  can have high content of tetraquarks

$a_0(980)$ ,  $K_0^*(800)$  can be dynamically generated states

Wolkanowski *et al.*, PRD 93, 014002; 1512.01071

$f_0(1500)$ ,  $f_0(1710)$  can have an admixture of glueballs

# Features of our approach

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  - $u, d, s$  constituent quarks ( $m_u = m_d$ )
  - Polyakov loop variables  $\Phi, \bar{\Phi}$  with  $\mathcal{U}_{\log}^{\text{YM}}$  or  $\mathcal{U}_{\log}^{\text{glue}}$
- no mesonic fluctuations included in the grand potential, only fermionic ones

$$\mathcal{Z} = e^{-\beta V \Omega(T, \mu_q)} = \int_{\text{PBC}} \prod_a \mathcal{D}\xi_a \int_{\text{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp \left[ - \int_0^\beta d\tau \int_V d^3x \left( \mathcal{L} + \mu_q \sum_f q_f^\dagger q_f \right) \right]$$

approximated as  $\Omega(T, \mu_q) = U_{\text{meson}}^{\text{tree}}(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(T, \mu_q) + \mathcal{U}_{\log}(\Phi, \bar{\Phi})$  with  $\tilde{\mu}_q = \mu_q - iG_4$

$$e^{-\beta V \Omega_{\bar{q}q}^{(0)}} = \int_{\text{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp \left\{ \int_0^\beta d\tau \int_x q_f^\dagger \left[ \left( i\gamma_0 \vec{\gamma} \cdot \vec{\nabla} - \frac{\partial}{\partial \tau} + \tilde{\mu}_q \right) \delta_{fg} - \gamma_0 \mathcal{M}_{fg} \Big|_{\xi_a=0} \right] q_g \right\}$$

- quarks not coupled to the (axial)vectors  $\Rightarrow$  tree-level (axial)vector masses
- fermionic vacuum and thermal fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model
- 4 coupled  $T/\mu_B$ -dependent field equations for condensates:  $\phi_N, \phi_S, \Phi, \bar{\Phi}$
- thermal contribution of  $\pi, K, f_0^L$  included in the pressure, however their curvature mass contains no mesonic fluctuations

# Effects of Polyakov loops on Fermi-Dirac statistics

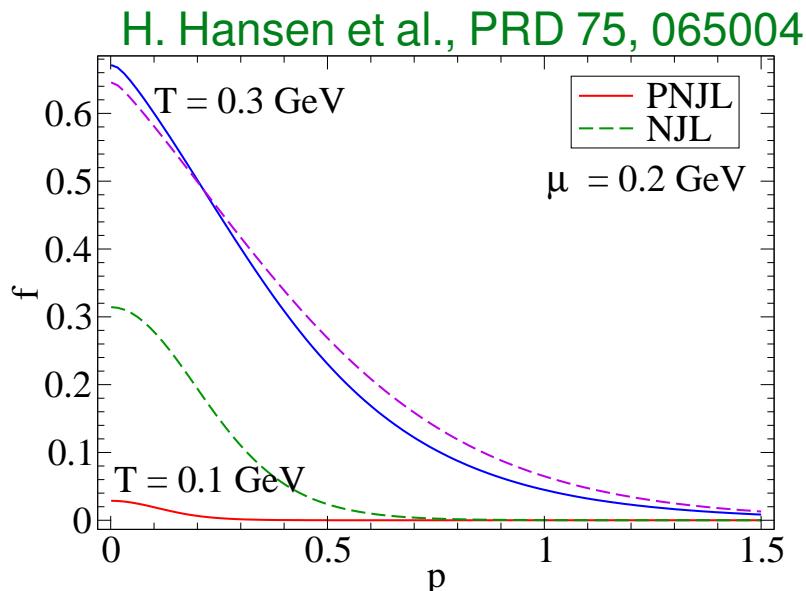
The inclusion of the Polyakov loop modifies the FD distribution function

$$f(E_p - \mu_q) \longrightarrow f_{\Phi}^{+}(E_p) = \frac{\left(\Phi + 2\bar{\Phi}e^{-\beta(E_p - \mu_q)}\right)e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}{1 + 3\left(\Phi + \bar{\Phi}e^{-\beta(E_p - \mu_q)}\right)e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}$$

$$f(E_p + \mu_q) \longrightarrow f_{\Phi}^{-}(E_p) = \frac{\left(\bar{\Phi} + 2\Phi e^{-\beta(E_p + \mu_q)}\right)e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}{1 + 3\left(\Phi + \bar{\Phi}e^{-\beta(E_p + \mu_q)}\right)e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}$$

$$\Phi, \bar{\Phi} \rightarrow 0 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(E_p \pm \mu_q)$$

**three-particle state appears:** mimics confinement of quarks within baryons



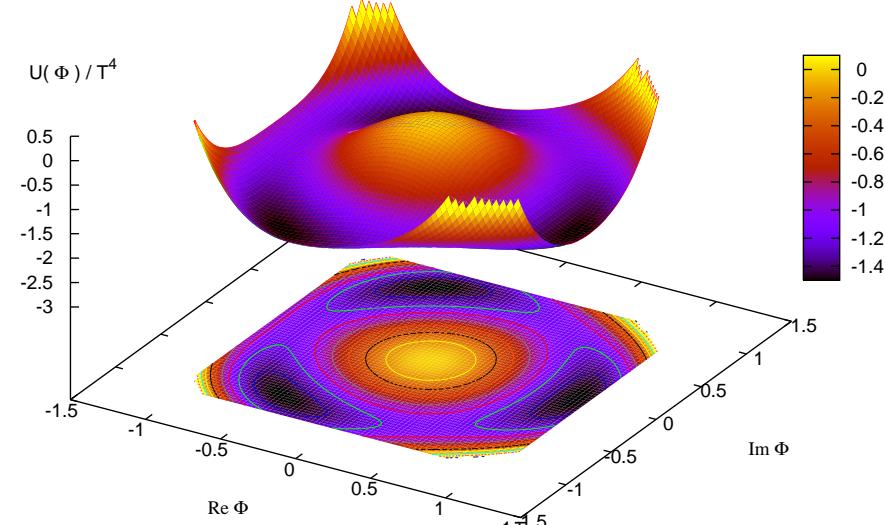
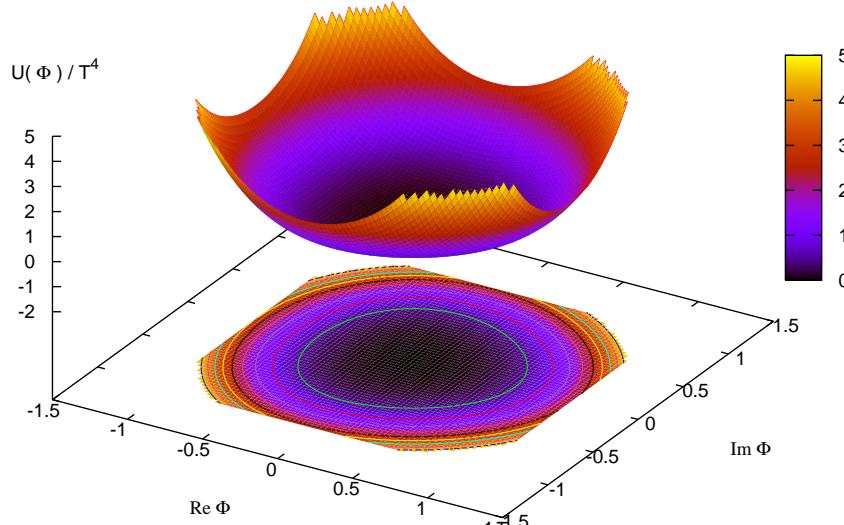
the effect of the Polyakov loop  
is more relevant for  $T < T_c$

at  $T = 0$  there is no difference between  
models with and without Polyakov loop:  
 $\Theta(3(\mu_q - E_p)) \equiv \Theta(\mu_q - E_p)$

# Polyakov loop potential

$T < T_0$  “Color confinement”  
 $\langle \Phi \rangle = 0 \rightarrow$  no breaking of  $\mathbb{Z}_3$   
 one minimum

$T > T_0$  “Color deconfinement”  
 $\langle \Phi \rangle \neq 0 \rightarrow$  spontaneous breaking of  $\mathbb{Z}_3$   
 minima at  $0, 2\pi/3, -2\pi/3$   
 one of them spontaneously selected



from Hansen et al., PRD 75, 065004

Logarithmic potential coming from the  $SU(3)$  Haar measure of group integration  
 K. Fukushima, Phys. Lett. B591, 277 (2004)

$$\frac{\mathcal{U}_{\log}^{\text{YM}}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T) \ln \left[ 1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2 \right]$$

$$a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3},$$

the parameters are fitted to the pure gauge lattice data

$a_0$	$a_1$	$a_2$	$b_3$
3.51	-2.47	15.2	-1.75

C. Ratti, et al., Eur. Phys. J. C 49, 213 (2007)

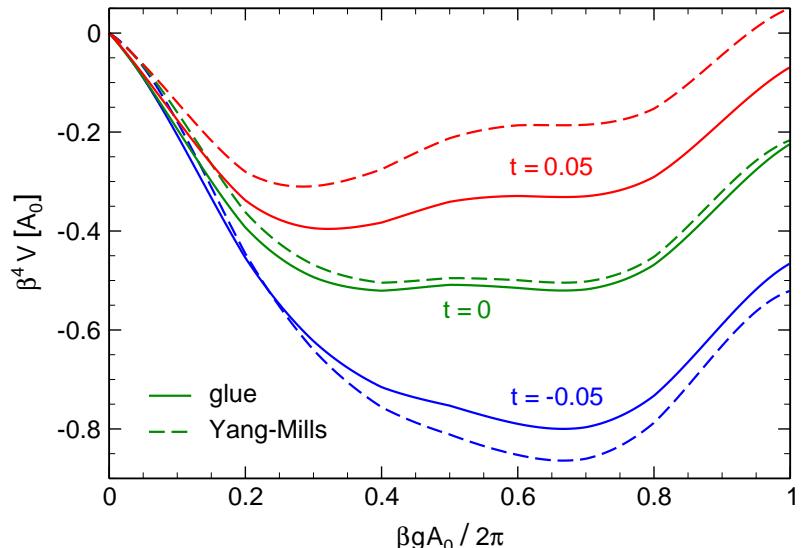
# Effect of quarks: Improved Polyakov loop potential

- $N_f$ -dependence of  $T_0$  estimated:  $T_0(N_f=2+1)=182$  MeV for  $m_s=95$  MeV  
L. M. Haas *et al.*, PRD 87, 076004 see also B.-J. Schaefer et al., PRD 76, 074023
- Using FRG, the glue potential  $\mathcal{U}^{\text{glue}}(\Phi, \bar{\Phi})$  coming from the gauge d.o.f. propagating in the presence of dynamical quarks can be matched to the potential  $\mathcal{U}^{\text{YM}}(\Phi, \bar{\Phi})$  of the  $SU(3)$  YM theory by relating the reduced temperatures:

$$\frac{\mathcal{U}^{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{\mathcal{U}^{\text{YM}}}{(T^{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}})) \quad \text{with} \quad t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}$$

$$t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T^{\text{YM}} - T_0^{\text{YM}}}{T_0^{\text{YM}}}, \quad T_c^{\text{glue}} \in (180, 270) \text{ MeV}$$

L. M. Haas *et al.*, PRD 87, 076004 (2013)



# Field equations (FEs)

Four coupled field equations are obtained by extremizing the grand potential

$$\Omega(T, \mu_q) = U_{\text{meson}}^{\text{tree}}(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)\text{vac}} + \Omega_{\bar{q}q}^{(0)T}(T, \mu_q) + \mathcal{U}_{\log}(\Phi, \bar{\Phi})$$

using  $\frac{\partial \Omega_H}{\partial \phi_N} = \frac{\partial \Omega_H}{\partial \phi_S} = \frac{\partial \Omega_H}{\partial \Phi} = \frac{\partial \Omega_H}{\partial \bar{\Phi}} = 0$   $E_f^\pm(p) = E_f(p) \mp \mu_q$ ,  $E_f^2(p) = p^2 + m_f^2$

$$1) - \frac{1}{T^4} \frac{dU(\Phi, \bar{\Phi})}{d\Phi} + \frac{6}{T^3} \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left( \frac{e^{-\beta E_f^-(p)}}{g_f^-(p)} + \frac{e^{-2\beta E_f^+(p)}}{g_f^+(p)} \right) = 0$$

$$2) - \frac{1}{T^4} \frac{dU(\Phi, \bar{\Phi})}{d\bar{\Phi}} + \frac{6}{T^3} \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left( \frac{e^{-\beta E_f^+(p)}}{g_f^+(p)} + \frac{e^{-2\beta E_f^-(p)}}{g_f^-(p)} \right) = 0$$

$$3) m_0^2 \phi_N + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - \frac{c_1}{\sqrt{2}} \phi_N \phi_S - h_{0N} + \frac{3}{2} g_F (\langle \bar{q}_u q_u \rangle_T + \langle \bar{q}_d q_d \rangle_T) = 0$$

$$4) m_0^2 \phi_S + (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - \frac{\sqrt{2}}{4} c_1 \phi_N^2 - h_{0S} + \frac{3}{\sqrt{2}} g_F \langle \bar{q}_s q_s \rangle_T = 0$$

renormalized fermion tadpole:

$$m_{u,d} = \frac{g_F}{2} \phi_N \quad \text{and} \quad m_s = \frac{g_F}{\sqrt{2}} \phi_S$$

$$\langle \bar{q}_f q_f \rangle_T = 4m_f \left[ -\frac{m_f^2}{16\pi^2} \left( \frac{1}{2} + \ln \frac{m_f^2}{M_0^2} \right) + \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_f(p)} (f_f^-(p) + f_f^+(p)) \right]$$

# Meson curvature masses

Used for parametrization and to calculate mesonic contribution to the pressure.

$i = S$  for scalar and  $i = P$  for pseudoscalar mesons:

$$\mathcal{M}_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_q)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = m_{i,ab}^2 + \Delta m_{i,ab}^2 + \delta m_{i,ab}^2$$

- $m_{i,ab}^2$  : tree-level mass squared matrix

- contribution of fermionic **vacuum** fluctuations

Chatterjee & Mohan, PRD85, 074018

$$\Delta m_{i,ab}^2 = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[ \left( \frac{3}{2} + \log \frac{m_f^2}{M_0^2} \right) m_{f,a}^{2(i)} m_{f,b}^{2(i)} + m_f^2 \left( \frac{1}{2} + \log \frac{m_f^2}{M_0^2} \right) m_{f,ab}^{2(i)} \right]$$

- contribution of fermionic **thermal** fluctuations

Schaefer & Wagner, PRD79, 014018  
Tiwari, PRD88, 074017

$$\delta m_{i,ab}^2 = 6 \sum_f \int_p \left[ \frac{f_f^+(p) + f_f^-(p)}{2E_f(p)} \left( m_{f,ab}^{2(i)} - \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2E_f^2(p)} \right) + \frac{B_f^+(p) + B_f^-(p)}{2E_f(p)} \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2TE_f(p)} \right]$$

where  $m_{f,a}^{2(i)} \equiv \frac{\partial m_f^2}{\partial \varphi_{i,a}}$ ,  $m_{f,ab}^{2(i)} \equiv \frac{\partial^2 m_f^2}{\partial \varphi_{i,a} \partial \varphi_{i,b}}$ ,  $E_f^2(p) = p^2 + m_f^2$ ,  $B_f^\pm(p)$  obtained from  $f_f^\pm(p)$

# Determination of the parameters

- set of 14+1 parameters –  $P = \{m_0^2, \lambda_1, \lambda_2, c_1, m_1^2, g_1, g_2, h_1, h_2, h_3, \delta_S, h_N \xleftrightarrow{FE} \phi_N, h_S \xleftrightarrow{FE} \phi_S, g_F\}$   $|P|=14$   
– the extra parameter,  $M_0 \in \{0.3, 0.9, 1.5\}$  GeV is kept fixed at first
- set  $O$  of observables  $|O| = 30$ 
  - 29 vacuum quantities
  - 15 masses
  - 8 curvature masses with fermionic corrections  
 $m_\pi, m_\eta, m_{\eta'}, m_K, m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}$
  - 5 tree-level (axial)vector masses  
 $m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}$
  - 2 tree-level constituent quark masses  $m_u, m_s$
  - 12 tree-level decay widths  $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^\star \rightarrow K\pi}$   
 $\Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^\star}$   
 $\Gamma_{a_0}, \Gamma_{K_0^\star \rightarrow K\pi}, \Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$
  - 2 PCAC relations involving  $f_\pi$  and  $f_K$
  - 1 medium related quantity:  $T_c$  at  $\mu_B = 0$
- parameters determined with a multiparametric  $\chi^2$  minimization  $\longrightarrow$  MINUIT

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{|O|} \left[ \frac{Q_i(\mathbf{p}) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2, \quad \mathbf{p} \in P \quad \text{used also in Paganlija et al., PRD 87, 014011}$$

$i^{\text{th}}$  observable:

- $Q_i(\mathbf{p}) \leftarrow$  value calculated within the model
- $Q_i^{\text{exp}} \leftarrow$  experimental value
- $\delta Q_i \leftarrow$  error (20% for scalars, 10% for quarks, 5% for the rest)

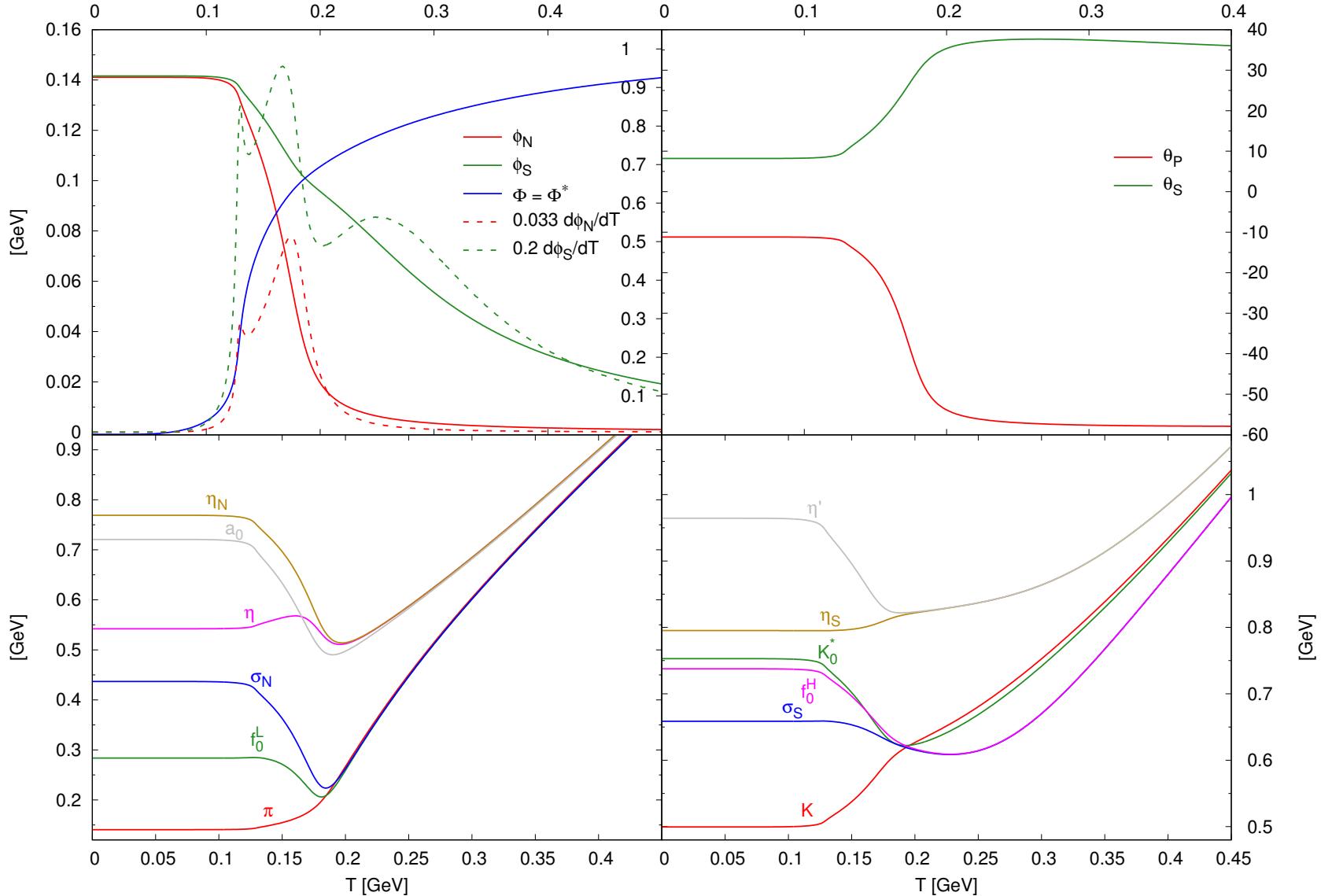
## Result of the parametrization

- 40 possible assignments of scalar mesons to the scalar nonet states
- 3 values of  $M_0$  are used  $\implies$  120 cases to investigate  
for each case  $5 \cdot 10^4 - 10^5$  configurations are used for the  $\chi^2$  minimization
- **lowest  $\chi^2$**  obtained for  $M_0 = 0.3$  GeV  $\chi^2 = 18.57$  and  $\chi^2_{\text{red}} \equiv \frac{\chi^2}{N_{\text{dof}}} = 1.16$   
**assignment:**  $a_0^{\bar{q}q} \rightarrow a_0(980)$ ,  $K_0^{*,\bar{q}q} \rightarrow K_0^*(800)$ ,  $f_0^{L,\bar{q}q} \rightarrow f_0(500)$ ,  $f_0^{H,\bar{q}q} \rightarrow f_0(980)$   
**problem:**  $m_{a_0} < m_{K_0^*}$ ,  $m_{f_0^{H/L}}$  too light
- by minimizing also for  $M_0$  we obtain using  $\mathcal{U}_{\log}^{\text{YM}}(\Phi, \bar{\Phi})$  with  $T_0 = 182$  MeV:

Parameter	Value	Parameter	Value
$\phi_N$ [GeV]	0.1411	$g_1$	5.6156
$\phi_S$ [GeV]	0.1416	$g_2$	3.0467
$m_0^2$ [GeV $^2$ ]	$2.3925E-4$	$h_1$	27.4617
$m_1^2$ [GeV $^2$ ]	$6.3298E-8$	$h_2$	4.2281
$\lambda_1$	-1.6738	$h_3$	5.9839
$\lambda_2$	23.5078	$g_F$	4.5708
$c_1$ [GeV]	1.3086	$M_0$ [GeV]	0.3511
$\delta_S$ [GeV $^2$ ]	0.1133		

The presented results are obtained with this set of parameters.

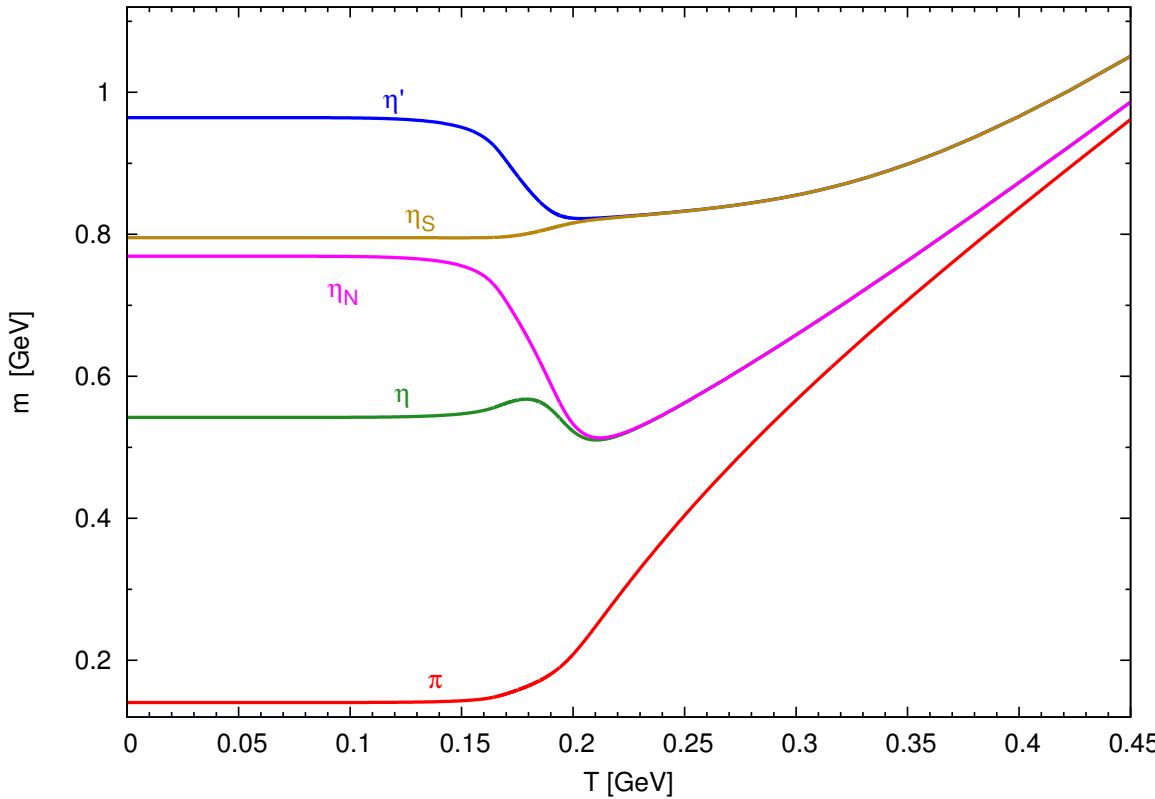
# $T$ -dependence of masses, condensates, and mixing angles



- chiral symmetry is restored at high  $T$ , as the chiral partners  $(\pi, f_0^L)$ ,  $(\eta, a_0)$  and  $(K, K_0^*)$ ,  $(\eta', f_0^H)$  become degenerate

- $U(1)_A$  symmetry is not restored, as the axial partners  $(\pi, a_0)$  and  $(\eta, f_0^L)$  do not become degenerate

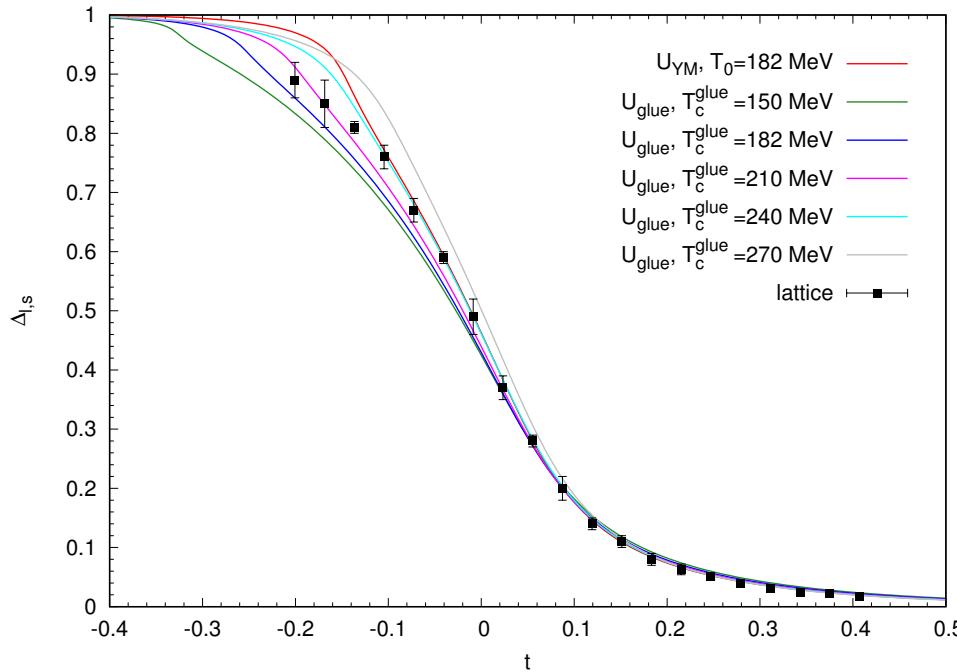
# Mass pattern in the $\eta, \eta'$ sector



- our pattern:  $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$  and similarly  $m_{f_0^L} \leq m_{\sigma_N} < m_{\sigma_S} \leq m_{f_0^H}$  and also  $a_0$  degenerates with  $\eta$   
in contrast to the pattern obtained w/o the inclusion of (axial)vector mesons  
Schaefer & Wagner, PRD79, 014018 (QM) and Tiwari, PRD88, 074017 (PQM)
- in the FRG study of Rennecke & Schaefer, arXiv:1610.08748 (w/o (axial)vector mesons))
  - LPA:  $a_0$ -meson degenerates with  $\eta'$ -meson
  - LPA'+Y:  $a_0$ -meson degenerates with  $\eta$ -meson

# $T$ -dependence of the condensates compared to lattice results

The subtracted chiral condensate



- lattice result shows a very smooth transition
- our result is completely off
- renormalization of the Polyakov loop could explain part of the discrepancy

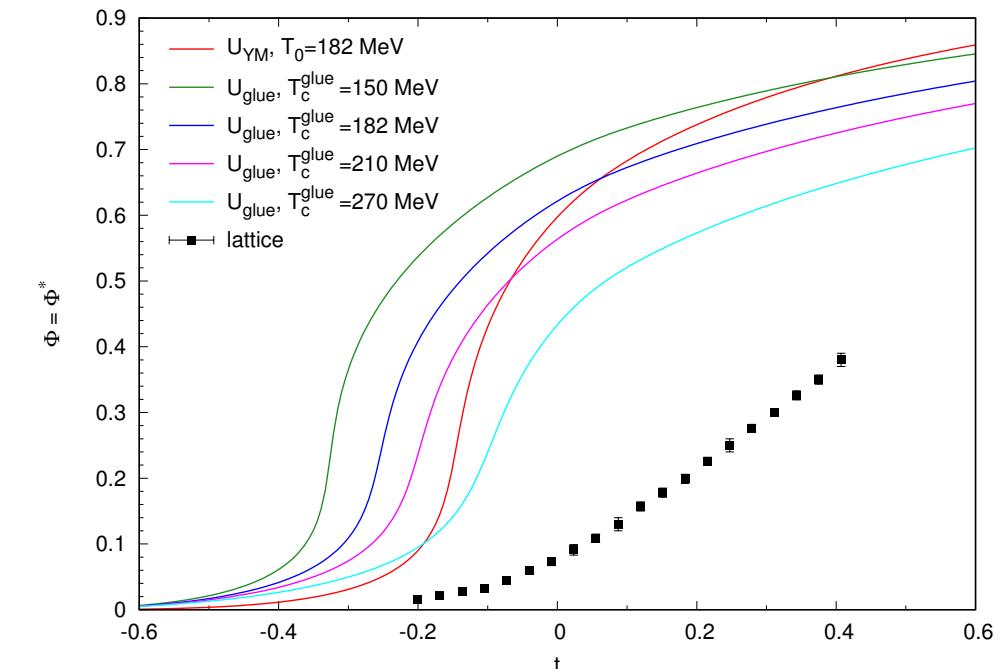
Andersen *et al.*, PRD92, 114504

– subtracted chiral condensate:

$$\Delta_{l,s} = \frac{\left( \Phi_N - \frac{h_N}{h_S} \cdot \Phi_S \right) \Big|_T}{\left( \Phi_N - \frac{h_N}{h_S} \cdot \Phi_S \right) \Big|_{T=0}}$$

–  $U_{\log}^{\text{glue}}$  with  $T_c^{\text{glue}} \in (210, 240)$  MeV gives good agreement with the lattice result of Borsányi *et al.*, JHEP 1009, 073 (2010)

Polyakov loop expectation values



## Thermodynamical observables

pressure:  $p(T, \mu_q) = \Omega_H(T=0, \mu_q) - \Omega_H(T, \mu_q)$

entropy density:  $s = \frac{\partial p}{\partial T}$

quark number density:  $\rho_q = \frac{\partial p}{\partial \mu_q}$

energy density:  $\epsilon = -p + Ts + \mu_q \rho_q$

scaled interaction measure:  $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$

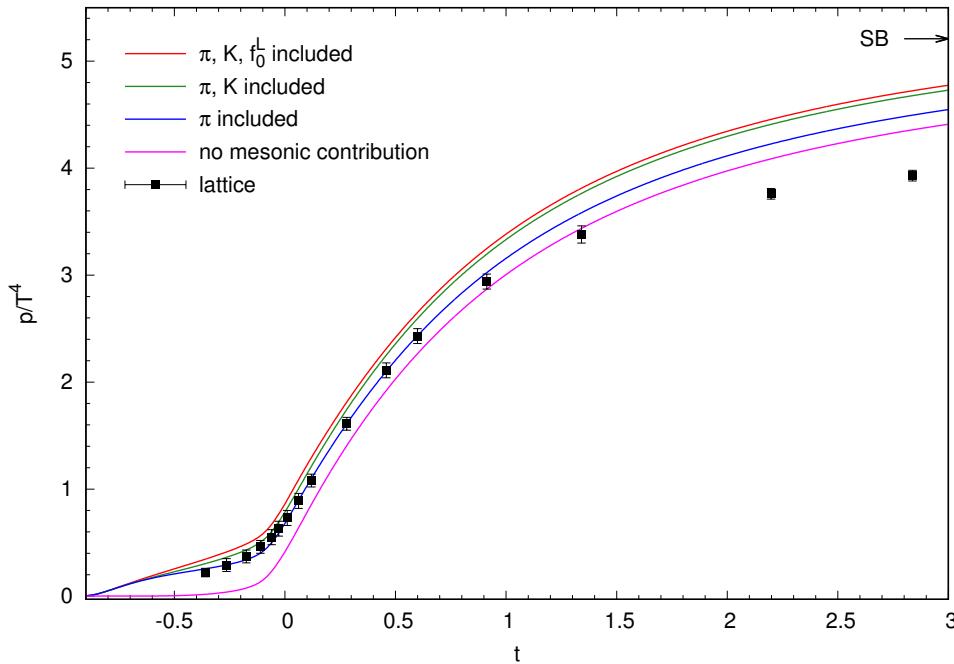
speed of sound at  $\mu_B = 0$ :  $c_s^2 = \frac{\partial p}{\partial \epsilon}$

We include **mesonic thermal contribution** to the **pressure**  $b \in \{\pi, K, f_0^L\}$

$$\Delta p_b(T) = -n_b T \int \frac{d^3 q}{(2\pi)^3} \ln(1 - e^{-\beta E_b(q)}), \quad E_b(q) = \sqrt{q^2 + m_b^2}$$

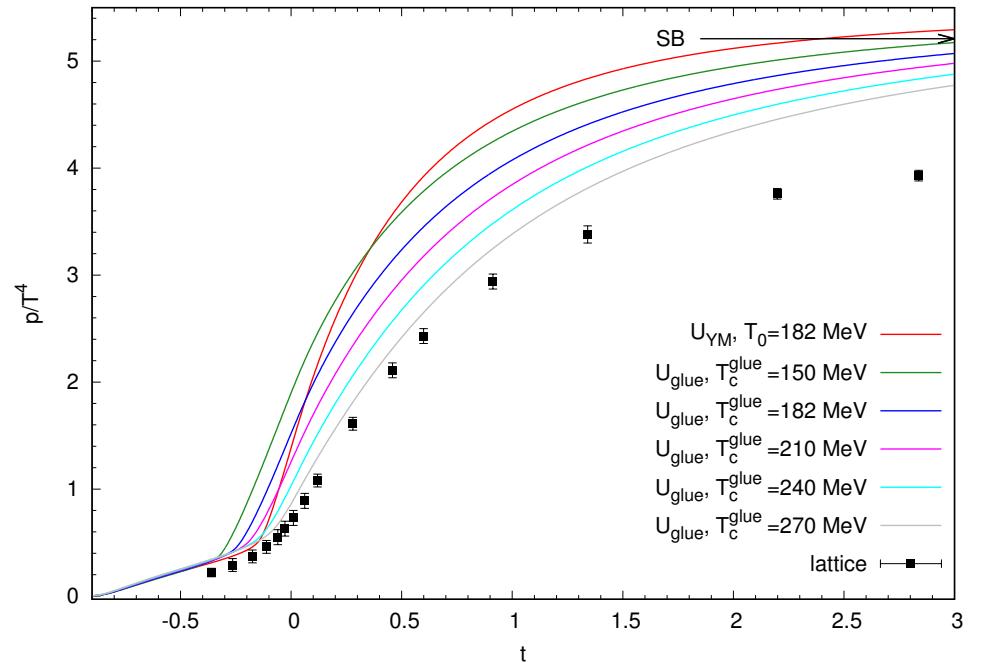
meson multiplicities:  $n_\pi = 3, n_K = 4, n_{f_0^L} = 1$

# Normalized pressure

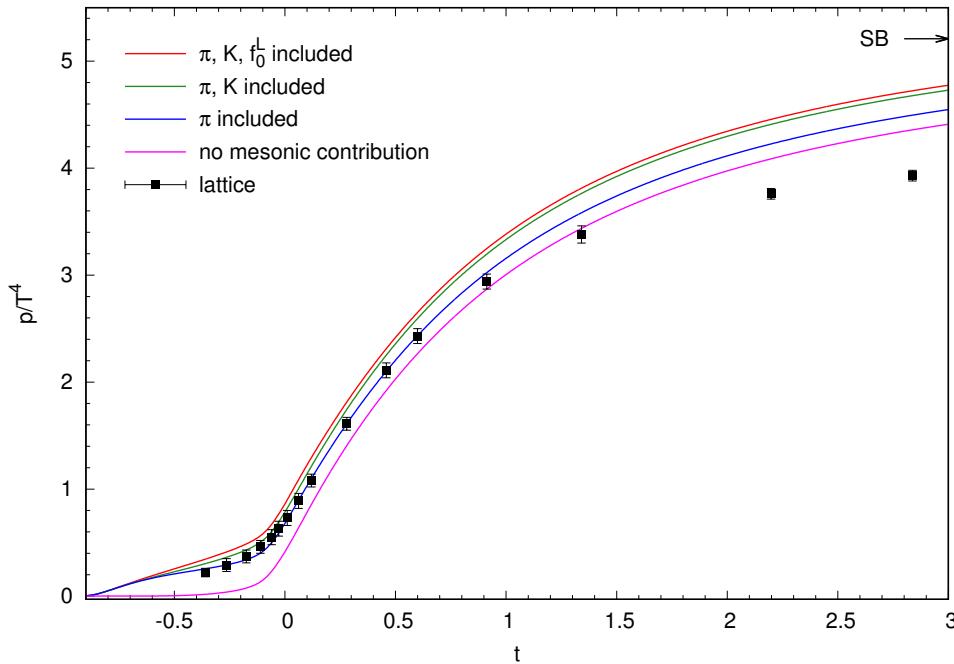


– overshooting increases with decreasing  $T_c^{\text{glue}}$

- we used  $U_{\text{glue}}$  with  $T_c^{\text{glue}} = 270 \text{ MeV}$
- pions dominate the pressure at small  $T$
- contribution of the kaons is important
- at high  $T$  the pressure overshoots the lattice data of [Borsányi et al., JHEP 1011, 077 \(2010\)](#)

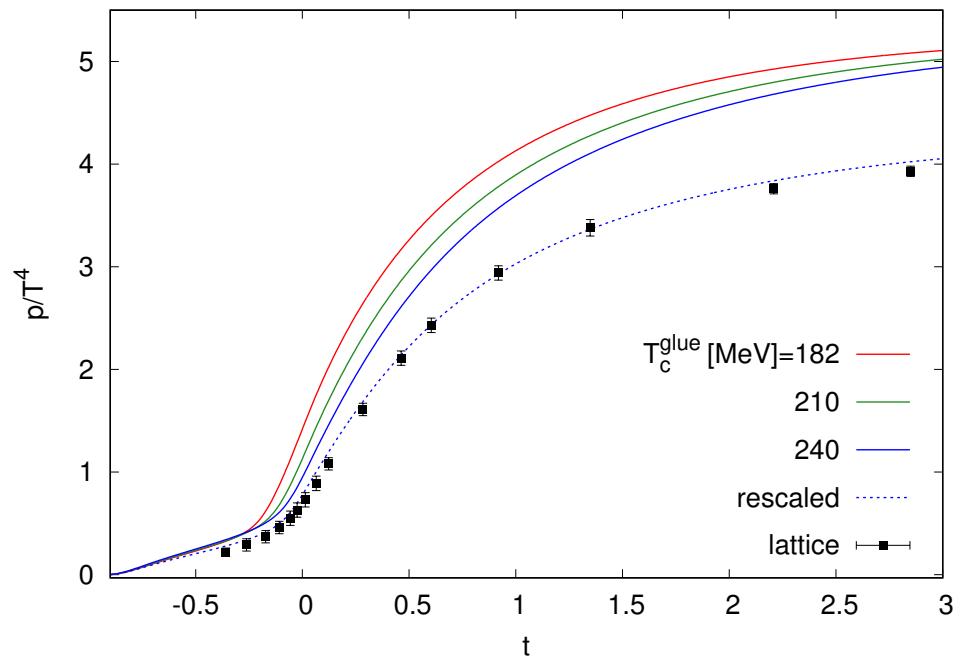


# Normalized pressure

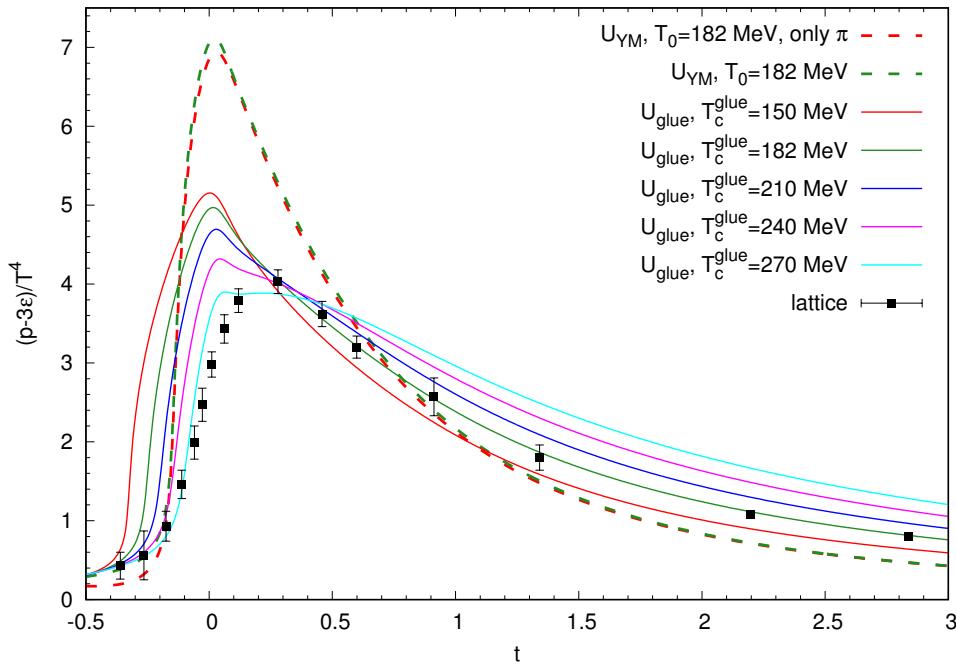


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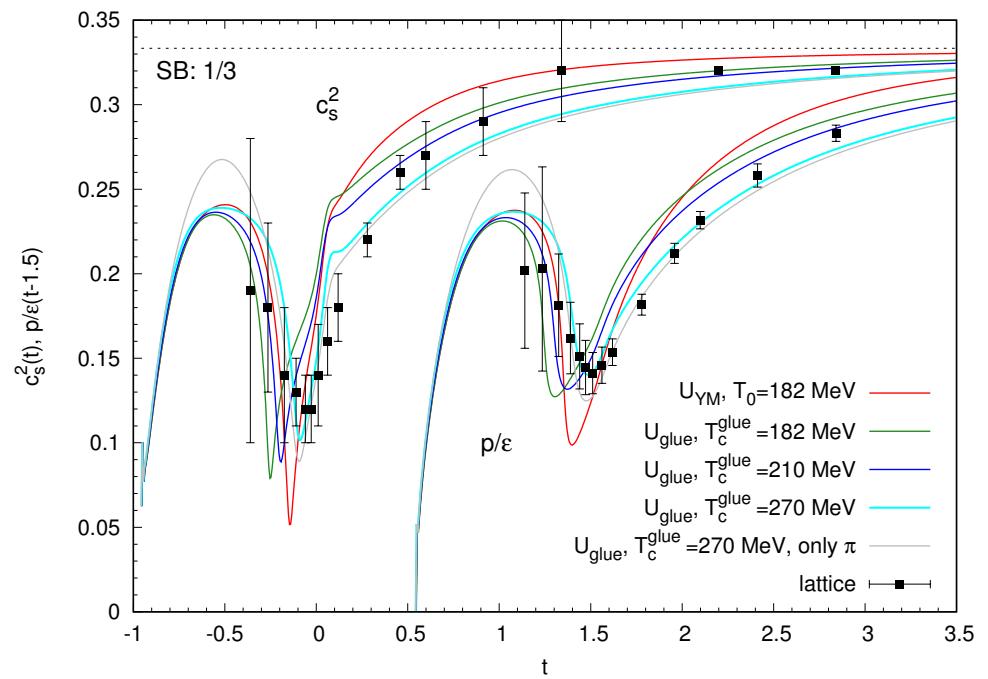
– scaling the pressure by a constant (0.82 for  $T_c^{\text{glue}} = 240 \text{ MeV}$ ) we almost recover the lattice data



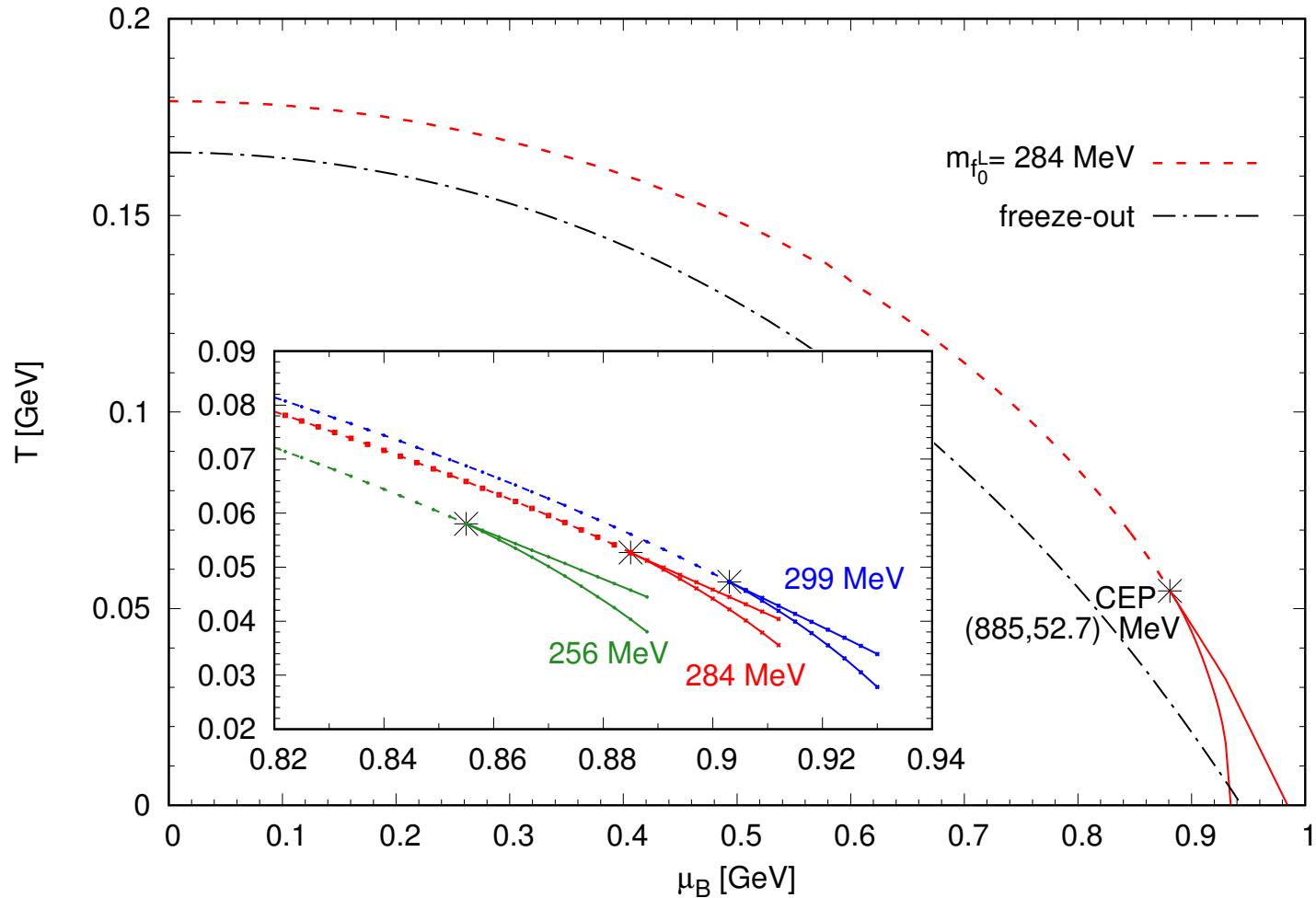
## Scaled interaction measure



## Speed of sound and $p/\epsilon$



# $T - \mu_B$ Phase Diagram



- we used  $U_{\log}^{\text{glue}}$  with  $T_c^{\text{glue}} = 210$  MeV
- freeze-out curve from Cleymans *et al.*, J.Phys.G 32, S165 (2006)
- curvature  $\kappa$  at  $\mu_B = 0$  obtained from the fit  $\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa \left( \frac{\mu_B}{T_c(\mu_B)} \right)^2$
- $\kappa = 0.0193$  obtained, close to the lattice value  $\kappa = 0.020(4)$  of Cea *et al.*, PRD93, 014507

## CEP obtained with other methods

- Dyson-Schwinger equation

$$(\mu_B, T)_{\text{CEP}} \approx (3.0, 0.9) T_c(\mu = 0)$$

Roberts *et al.*, PRL106 (2011) 172301

$$(\mu_B, T)_{\text{CEP}} \approx (660, 97) \text{ MeV for } N_f = 2 + 1$$

Gutierrez *et al.*, J.Phys. G41 (2014) 075002

$$(\mu_B, T)_{\text{CEP}} = (504, 115) \text{ MeV for } N_f = 2 + 1$$

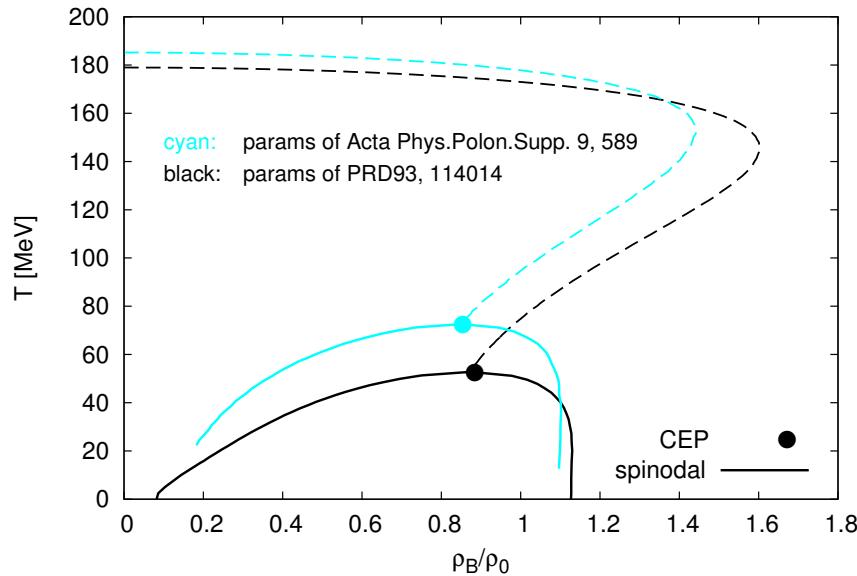
Fischer *et al.*, PRD90 (2014) 034022

- FRG study of Rennecke & Schaefer, arXiv:1610.08748

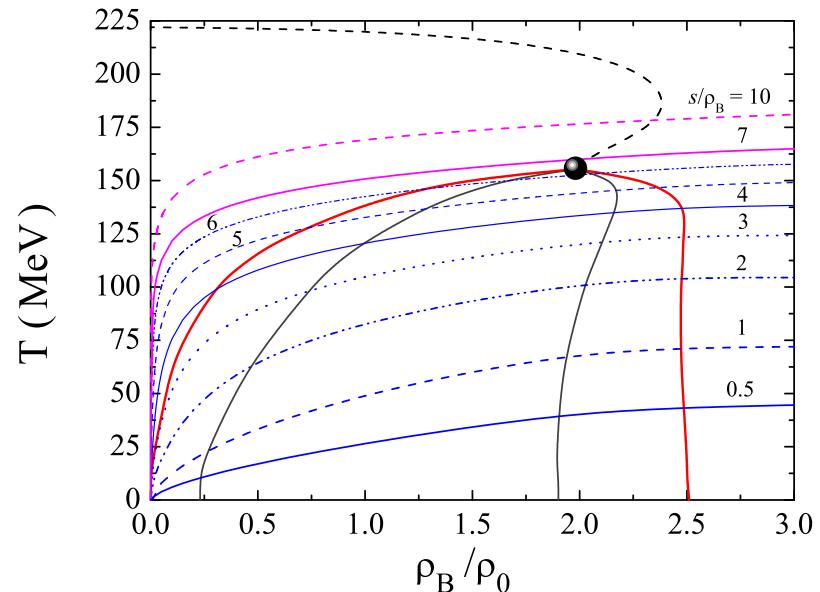
$$\begin{aligned} (\mu_B, T)_{\text{CEP}} = & \quad (795, 44) \text{ MeV} && \text{LPA} \\ & (765, 46) \text{ MeV} && \text{LPA} + Y \\ & (705, 61) \text{ MeV} && \text{LPA}' + Y \end{aligned}$$

# $T - \rho_B$ Phase Diagram

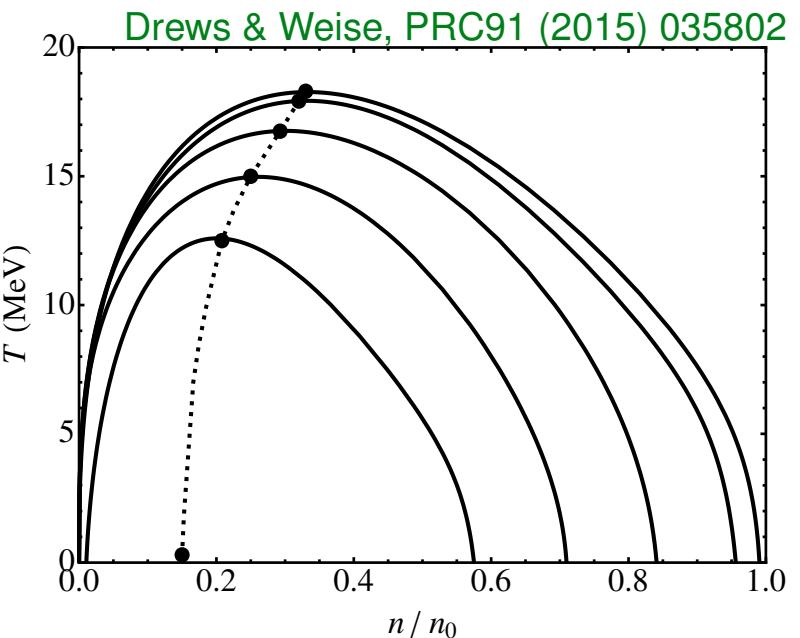
our model



PNJL with vector interaction  
 P. Costa, PRD93 (2016) 114035

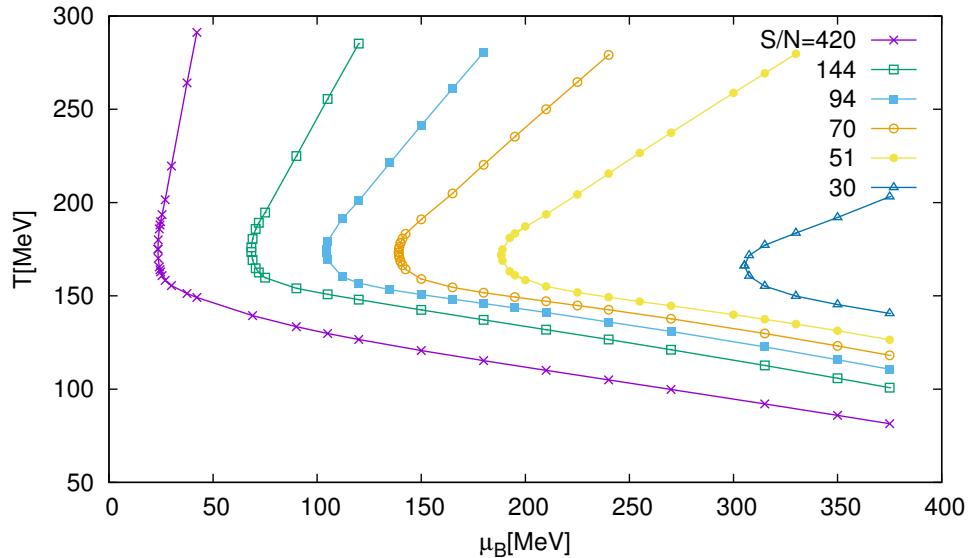


- the  $T - \rho_B$  phase diagram of our model is closer to that of the nuclear liquid-gas PT than to those obtained in other chiral models

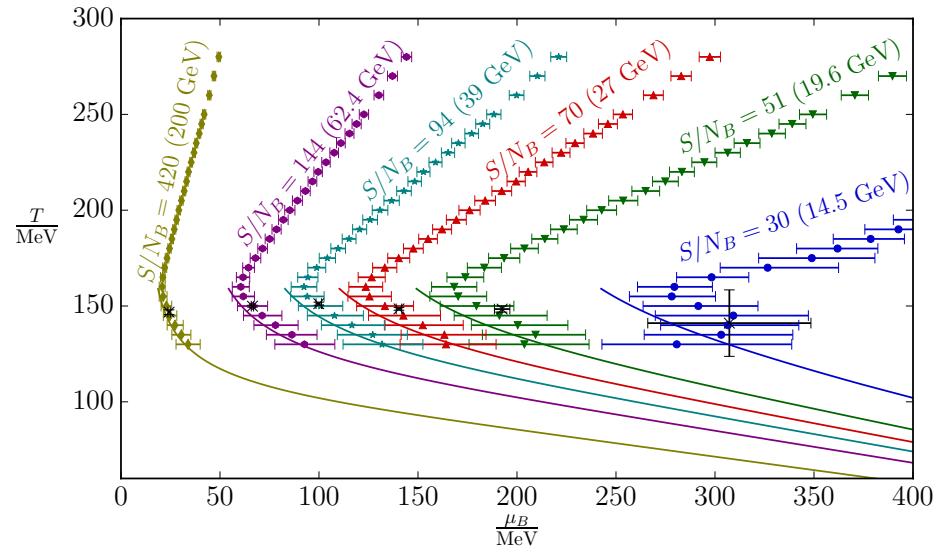


# Isentropic trajectories in the $T - \mu_B$ plane

our model, where  $\mu_B^{\text{CEP}} > 850\text{MeV}$



lattice (analytic continuation)  
Günther *et al.*, arXiv:1607.02493

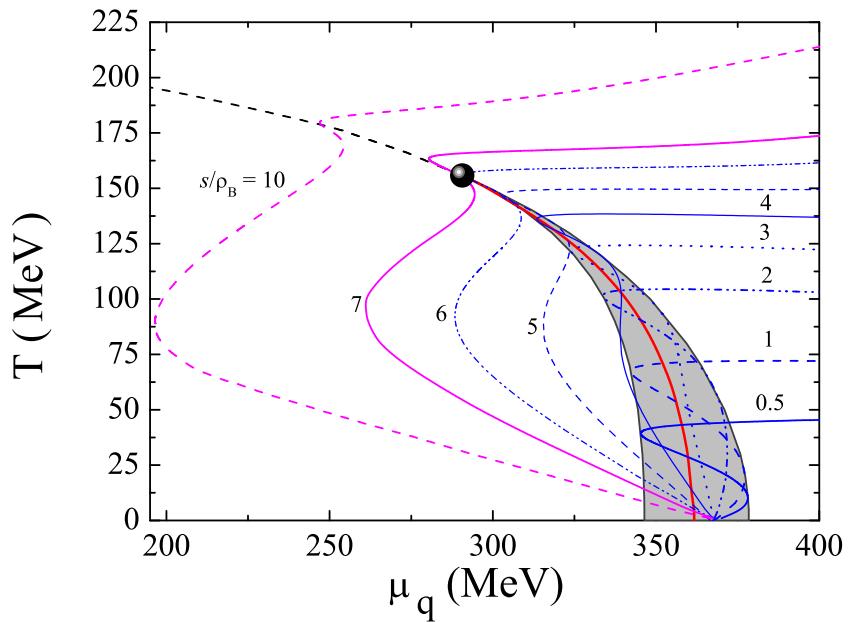


- same qualitative behavior of the isentropic trajectories for  $\mu_B \leq 400$  MeV  
 $\implies$  indication that in the lattice result there is no CEP in this region of  $\mu_B$

# Isentropic trajectories in the $T - \mu_B$ plane

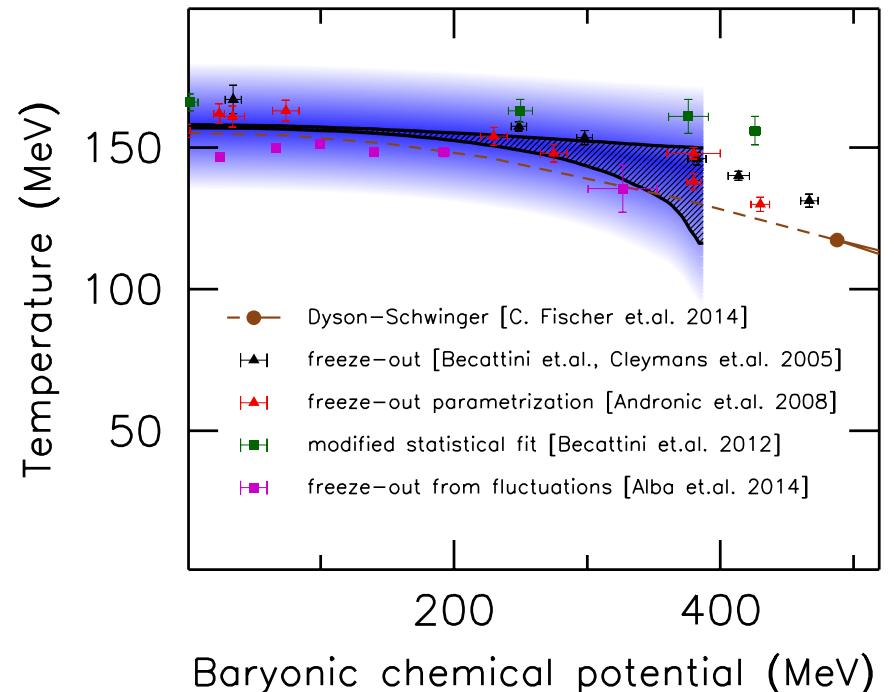
PNJL with vector interaction

P. Costa, PRD93 (2016) 114035



lattice (analytic continuation)

Bellwied *et al.*, PLB751 (2015) 559



- effective models show a different behavior of the isentropic trajectories close to CEP compared to the small  $\mu_B$  case
- no indication for CEP from the continuum extrapolated lattice results analytically continued to the  $\mu_B \leq 400$  MeV region

## Summary and Conclusions

- The thermodynamics of the PQM model including (axial)vectors was studied after parameterizing the model with a modification of the method used in  
*Paganlja et al.*, PRD 87, 014011.
- 40 possible assignments of scalars to the nonet states were investigated.  
Lowest  $\chi^2$  for  $a_0^{\bar{q}q} \rightarrow a_0(980)$ ,  $K_0^{*,\bar{q}q} \rightarrow K_0^*(800)$ ,  $f_0^{L,\bar{q}q} \rightarrow f_0(500)$ ,  $f_0^{H,\bar{q}q} \rightarrow f_0(980)$ .
- For the best set of parameters a CEP was found in the  $\mu_B - T$  plane.  
A self-consistent treatment of quarks will most probably decrease  $\mu_B^{\text{CEP}}$  and increase  $T_c^{\text{CEP}}$ .
- $T$  and  $\mu_B$  dependence of various thermodynamical observables measured on the lattice is qualitatively reproduced with an improved Polyakov loop potential.
- From a comparison of the isentroping courses with those measured on the lattice it seems unlikely to have  $\mu_B^{\text{CEP}} < 400 \text{ MeV}$ .
- The model and the approximation used to solve it should be improved by:
  - including tetraquarks for a more reliable vacuum phenomenology;
  - coupling the constituent quarks to the (axial)vectors;
  - including mesonic fluctuations;
  - treating the quarks self-consistently.